Abstract—Hyperspectral unmixing aims at identifying the unique spectral signatures (endmembers) and their corresponding proportions (abundances) from an observed hyperspectral scene. Many existing hyperspectral unmixing algorithms were developed rely on assumptions of pure pixel existence. However, the pure pixel assumption has high probability to seriously violated for highly mixed data. Chan's incorporate convex analysis and Craig's criterion to develop a minimum-volume enclosures implex (MVES) formulation for hyperspectral unmixing which estimates the endmembers by vertices of a minimum-volumes implex enclosing all the observed pixels. A cyclic minimization algorithm for approximating the MVES problem is developed using linear programs (LPs), which can be practically implemented by readily available LP solvers.

Index terms: Minimum Volume Enclosing Simplex, Hyperspectral Imaging, Signature Matrix, Abundance Maps.

I. INTRODUCTION

Hyperspectral imaging collects and processes information from across the electromagnetic spectrum. With correct extraction, hyperspectral imaging is able to describe material composition of observed object. When the hyperspectral scene is capture surfaces, each pixel of the observed spectra usually comprises multiple spectral signatures (endmembers) due to low spatial resolution of the sensor used.

Endmember extraction is to determine the endmembers that contribute to the measured spectra that form hyperspectral image. A number of hyperspectral extraction especially with assumption of existence of pure pixels, pixels that are contributed by a single endmember, in the observed data set, e.g., N-finder[1], vertex component analysis (VCA) [2], and Pure Pixel Index (PPI) [3]. These algorithm attempt to search for the purest observed pixel over the data set. However, pure pixel assumptions maybe seriously violated for the case of highly mixed data.

In this paper, we use Minimum Volume Enclosing Simplex (MVES) proposed by Chan et al [4] to implement hyperspectral unmixing without pure pixel assumption. The algorithm implementation is performed by dimension reduction of the observed pixels through a convex analysis concept called affine set fitting [5] continued by Craig's unmixing criterion [6] to formulate the hyperspectral unmixing as a MVES optimization problem.

This paper is organised as follow. The general explanation about problem and its assumptions are described in section II. Concepts of convex analysis related on unmixing process are explained in section III. Section IV explains MVES algorithm proposed by Chan et al. Section V presents simulation results based on two scenario, first scenario was using sintetic data generated with dirichlet distribution[7] associated with existing spectral signature from USGS spectral library and the last scenario was using data which had been taken from AVIRIS instrument[8].

Before advancing to next section, we presented list of notation we used:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>$i$th pixel</td>
</tr>
<tr>
<td>$s_n$</td>
<td>Signature matrix of $n$th endmember</td>
</tr>
<tr>
<td>$s_n(i)$</td>
<td>Signature matrix of $n$th endmember on $i$th pixel</td>
</tr>
<tr>
<td>$a_n$</td>
<td>$n$th abundance map</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of endmembers</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of pixels</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Endmember weight</td>
</tr>
<tr>
<td>$C^\dagger$</td>
<td>Pseudo-inverse of $C$ matrix</td>
</tr>
<tr>
<td>$\mathbb{R}, \mathbb{R}^N, \mathbb{R}^{M \times N}$</td>
<td>Real number set, N real number vector and $M \times N$ matrix.</td>
</tr>
</tbody>
</table>
\[ \mathbb{R}, \mathbb{R}_N^+, \mathbb{R}_M^{N\times N} \] Positive real number set, \( N \) positive real number vector and \( M \times N \) positive matrix.

\[ 1_N \] Vektor sebanyak-\( N \) dengan nilai homogen = 1

\[ I_N \] Matrik indentitas dengan ukuran \( N \times N \)

**II. PROBLEM AND ASSUMPTIONS**

Consider a scenario where a hyperspectral sensor explores a scene of interest where \( N \) different and unknown substances are involved on \( M \) spectral bands.

Imagine a hyperspectral image as a cube of images which each pixel vector can be described as a linear mixing model:

\[
x[n] = As[n] = \sum_{i=1}^{N} s_i[n] a_i, n = 1, \ldots, L
\]  

(1)

Where \( x[n] = [x_1[n], \ldots, x_M[n]]^T \) represents \( n \)th pixel comprising \( M \) spectral bands, \( A = [a_1, \ldots, a_N] \in \mathbb{R}^{M\times N} \) denotes the signature matrix whose \( i \)th column vector \( a_i \) is the \( i \)th endmember signature, \( s[n] = [s_1[n], \ldots, s_N[n]]^T \in \mathbb{R}^N \) denotes an abundance vector comprising \( N \) fractional abundances, and \( L \) is the total number of observed pixel vectors.

Each observed pixel is a linear combination of endmember signatures, \( A \), weighted by its abundance fractions, \( s[n] \). Figure 1 illustrates the linear spectral mixing model for hyperspectral imaging.

The goal of endmember unmixing is to estimate the signature matrix \( A \) and the abundances \( s[n] \) from the observed pixel \( x[1], \ldots, x[L] \) without prior knowledge about \( A \) and \( s[n] \). MVES algorithm assumes the number of endmembers to be known on the beginning. MVES algorithm is based on the following general assumptions:

A1. (Nonnegativity condition) for all \( i = 1, \ldots, N \) and \( n = 1, \ldots, L, s_i[n] \geq 0 \).

A2. (Full additivity condition) for all \( n = 1, \ldots, L, \sum_{i=1}^{N} s_i[n] = 1 \).

A3. \( \min\{L, M\} \geq N \) and \( A \) is full column rank.

**III. BASIC CONVEX ANALYSIS**

**A. Affine Hull**

Given a set of vectors \( \{a_1, \ldots, a_N\} \subset \mathbb{R}^M \), then the affine hull of \( \{a_1, \ldots, a_N\} \) is defined as

\[
\text{aff} \{a_1, \ldots, a_N\} = \{x = \sum_{i=1}^{N} \theta_i a_i | \theta \in \mathbb{R}^N, 1\}
\]

(2)

where \( \theta = [\theta_1, \ldots, \theta_N] \). An affine hull is an affine set, therefore can be represented as

\[ \text{Figure 1. Illustration of linear mixing model of hyperspectral image} \]
where \( A(.,.) \) denotes an affine set parameterized by a 2-tuple \((C, d) \in \mathbb{R}^{M \times N} \times \mathbb{R}^M\) with \(\text{rank}(C) = P\).

Here \( P \) is the affine dimension of \( \text{aff} \{a_1, \ldots, a_N\} \) and must satisfy \( P \leq N - 1 \). If \( \{a_1, \ldots, a_N\} \) is affinely independent which means that \( a_1 - a_N, \ldots, a_{N-1} - a_N \) are linearly independent, then \( P = N - 1 \).

To answer the problem about finding affine set parameter \((C, d)\), given \(\{a_1, \ldots, a_N\}\) and \(P\). For general case where \(p < N - 1\), \((C, d)\) can be found by solving the following affine set problem:

\[
\arg\min_{(C, d)} \sum_{i=1}^{N} e_{A(C,d)}(a_i)
\]

where \( e_{A}(a_i) \) is the projection error of \( a_i \) onto the set \(A\) and defined as

\[
e_{A}(a_i) = \min_{a \in A} \|a_i - a\|^2
\]

Constraint \(C^T C = I_p\) is to restrict \(C\) to have \(\text{rank}(C) = P\). It is shown that problem (4) has simple closed-form solution [9] given by:

\[
d = \frac{1}{N} \sum_{i=1}^{N} a_i
\]

\[
C = [q_1(UU^T), q_2(UU^T), \ldots, q_p(UU^T)]
\]

where \( U = [a_1, a_2, \ldots, a_N - d] \in \mathbb{R}^{M \times N} \) and \( q_i(R) \) is the eigenvector associated with the \(i\)th principal of the square matrix \(R\).

**B. Convex Hull**

Given set of vectors \(\{a_1, \ldots, a_N\} \subset \mathbb{R}^N\), then the convex hull of \(\{a_1, \ldots, a_N\}\) is defined as

\[
\text{conv}\{a_1, \ldots, a_N\} = \left\{ x = \sum_{i=1}^{N} \theta_i a_i \mid \theta_i \in \mathbb{R}_+^N, \sum_{i=1}^{N} \theta_i = 1 \right\}
\]

A convex hull is called simplex if \(M = N - 1\) and \(\{a_1, \ldots, a_N\}\) is affinely independent. A point \(x \in \text{conv}\{a_1, \ldots, a_N\}\) is a vertex of \(\text{conv}\{a_1, \ldots, a_N\}\) if \(x \neq \sum_{i=1}^{N} \theta_i a_i\) for all \(\theta \in \mathbb{R}_+^N\) and \(\sum_{i=1}^{N} \theta_i = 1\) and \(\theta \neq e_i\) for any \(i\). For general convex hull the set of all of its vertices is a subset of \(\{a_1, \ldots, a_N\}\), however for a simplex the set is exactly \(\{a_1, \ldots, a_N\}\).

**IV. MINIMUM VOLUME ENCLOSING SIMPLEX ALGORITHM**

Considering a signal model in (1) under second assumption we can infer that

\[
x[n] \in \text{aff} \{a_1, \ldots, a_N\}, \forall n
\]

In addition we can recover the affine hull of observed pixels \(x[1], x[2], \ldots, x[L]\) with considering the following lemma.

**Lemma 1 (Affine Hull Consistency)**

\[
\text{aff} \{x[1], \ldots, x[L]\} = \text{aff} \{a_1, \ldots, a_N\}
\]

Since \(a_1, \ldots, a_N\) are affinely independent as assumed in A3, the endmember affine hull \(\text{aff} \{a_1, \ldots, a_N\}\) can be represented as

\[
\text{aff} \{a_1, \ldots, a_N\} = \{x = Ca + d \mid \alpha \in \mathbb{R}^{N-1}\}
\]

\[
= A(C, d)
\]

for \(A(C, d) \in \mathbb{R}^{M \times (N-1)} \times \mathbb{R}^M\) and \(\text{rank}(C) = N - 1\).

From Lemma 1 and (11), the affine hull parameter \((C, d)\) for both \(\{a_1, \ldots, a_N\}\) and \(\text{aff} \{x[1], \ldots, x[L]\}\) can be estimated through affine set fitting as follows:

\[
d = \frac{1}{L} \sum_{n=1}^{L} x[n]
\]

\[
C = [q_1(UU^T), q_2(UU^T), \ldots, q_{N-1}(UU^T)]
\]

where \( U = [x[1] - d, \ldots, x[L] - d] \in \mathbb{R}^{M \times (N-1)}\).

Since \(x[n] \in A(C, d)\) and affine hull is a combination of linearity operation we can infer that

\[
x[n] = C\tilde{x}[n] + d
\]

where \(\tilde{x}[n]\) is the inverse image of \(x[n]\).

\[
\tilde{x}[n] = C^T(x[n] - d) \in \mathbb{R}^{N-1}
\]
Substituting (1) with (15) will produce
\[ \tilde{x}[n] = \sum_{j=1}^{N} s_j[n](C^\dagger a_j - C^\dagger d) \]
according to second assumption (A2) we can infer that dimension reduced pixel \( \tilde{x}[1], \ldots, \tilde{x}[L] \) can be inferred as
\[ \tilde{x}[n] = \sum_{j=1}^{N} s_j[n](C^\dagger a_j - C^\dagger d) = \sum_{j=1}^{N} s_j[n]\alpha_j \]
where
\[ \alpha_j = C^\dagger a_j - C^\dagger d \in \mathbb{R}^{N-1} \]
Is the \( j \)th dimension-reduced endmember. (18) also contribute to state the following lemma

**Lemma 2 (Simplex Geometry)**
\[ \tilde{x}[n] \in \text{conv}\{\alpha_1, \ldots, \alpha_n\} \subset \mathbb{R}^{N-1}, \forall n \]
and \( \text{conv}\{\alpha_1, \ldots, \alpha_n\} \) is a simplex for \( \mathbb{R}^{N-1} \).

Lemma 2 implies that all the dimension reduced pixels \( \tilde{x}[1], \ldots, \tilde{x}[L] \) must be inside the simplex constructed by the dimension reduced endmembers \( \alpha_i \) for \( i = 1, \ldots, N \). \( \tilde{x}[1], \ldots, \tilde{x}[L] \) also can be enclossed by a different simplex, other than \( \text{conv}\{\alpha_1, \ldots, \alpha_n\} \), denoted with \( \text{conv}\{\beta_1, \ldots, \beta_N\} \) as seen in figure 2. To sum up, can be expected that the data enclosing simplex with the minimum volume should coincide with the true endmember simplex \( \text{conv}\{\alpha_1, \ldots, \alpha_n\} \).

**IV.A. MVES Problem for hyperspectral unmixing**

Problem of finding MVES can be formulated as:
\[ \min_{\beta_1, \ldots, \beta_N} V(\beta_1, \ldots, \beta_N) \]
\[ \tilde{x}[n] \in \text{conv}\{\beta_1, \ldots, \beta_N\}, \forall n \]
where \( V(\beta_1, \ldots, \beta_N) \) is the volume of simplex \( \text{conv}\{\beta_1, \ldots, \beta_N\} \subset \mathbb{R}^{N-1} \) inferred as
\[ V(\beta_1, \ldots, \beta_N) = \frac{|\det(\Delta(\beta_1, \ldots, \beta_N))|}{(N-1)!} \]
\[ \Delta(\beta_1, \ldots, \beta_N) = \begin{bmatrix} \beta_1 & \ldots & \beta_N \\ 1 & \ldots & 1 \end{bmatrix} \]

With additional assumption

A4. There exist at least one index set \( \{l_1, l_2, \ldots, l_N\} \) such that \( \tilde{x}[l_i] = \alpha_i \) for \( i = 1, \ldots, N \).

Constraint for (20) can be described that
\[ \text{conv}\{\tilde{x}[1], \ldots, \tilde{x}[L]\} \subseteq \text{conv}\{\beta_1, \ldots, \beta_N\} \]
under 4th assumption we can have
\[ \text{conv}\{\tilde{x}[1], \ldots, \tilde{x}[L]\} = \text{conv}\{\tilde{x}[l_1], \ldots, \tilde{x}[l_N]\} = \text{conv}\{\alpha_1, \ldots, \alpha_N\} \]
Hence (24) becomes
\[ \text{conv}\{\alpha_1, \ldots, \alpha_N\} \subseteq \text{conv}\{\beta_1, \ldots, \beta_N\} \]
that means \( \alpha_i \in \text{conv}\{\beta_1, \beta_2, \ldots, \beta_N\} \). Then \( \alpha_i \) can be inferred as
\[ \alpha_i = \sum_{j=1}^{N} \theta_j \beta_j \]
where \( \sum_{j=1}^{N} \theta_j = 1 \) and \( \theta_j \geq 0 \) for \( i = 1, \ldots, N \).

From (23) and (26) can be inferred that
\[ \Delta(\alpha_1, \ldots, \alpha_N) = \Delta(\beta_1, \ldots, \beta_N) \Theta^T \]
where $\Theta = [\theta_{ij}] \in \mathbb{R}^{N \times N}$. By (27) and (22) can be inferred that

$$V(\alpha_1, \ldots, \alpha_N) = \frac{|\det(\Delta(\alpha_1, \ldots, \alpha_N))|}{(N-1)!}$$ (28)

According on lemma 1 reported in [11], can be stated $|\det(\Theta)| \leq 1$ and the equality holds if and only if $\Theta$ is permutation matrix. Can be concluded that

$$V(\alpha_1, \ldots, \alpha_N) \leq V(\beta_1, \ldots, \beta_N)$$ (29)

This further implies that the optimum solution for $\{\beta_1, \ldots, \beta_N\}$ is $\{\alpha_1, \ldots, \alpha_N\}$.

IV.B. MVES Algorithm

An alternative cost function in (20) is given by [10]

$$V(\beta_1, \ldots, \beta_N) = \frac{|\det(B)|}{(N-1)!}$$ (30)

For any dimension reduced pixels

$$\tilde{x}[n] = \sum_{i=1}^{N} s_i[n] \beta_i = \beta_N + Bs'[n]$$ (32)

where $s'[n] = (s_1[n], \ldots, s_{N-1}[n]) = 0$ and $s_N[n] = 1 - 1^T_N s'[n] \geq 0$. Then (20) is equivalent to

$$\min |\det(B)|$$

$$B, \beta_N$$

$$s'[1], \ldots, s'[L]$$ (33a)

With constraint

$$s'[n] \geq 0, 1^T_N s'[n] \leq 1$$

$$\tilde{x}[n] = \beta_N + Bs'[n], \forall n = 1, \ldots, L$$ (33b)

(33a) is still nonconvex and nonlinearity of (33b) need to be reformulated. Consider on to one mapping of the optimization variables:

$$H = B^{-1} \in \mathbb{R}^{(N-1) \times (N-1)}$$ (34a)

$$g = B^{-1} \beta_N \in \mathbb{R}^{(N-1)}$$ (34b)

Then $s'[n]$ can be represented as

$$s'[n] = B^{-1}(\tilde{x}[n] - \beta_N) = H\tilde{x}[n] - g$$ (35)

substituting (34) and (35) into (33), (33) can be reformulated with

$$\max |\det(H)|$$

$$H, g$$

$$H\tilde{x}[n] - g \geq 0$$

$$1^T_N (H\tilde{x}[n] - g) \leq 1, \forall n = 1, \ldots, L$$ (36b)

(36) has convex feasible set but its objective is still nonconvex. Chan et al used efficient LP to tackle the problem.

$$\det(H) = \sum (-1)^{i+j} h_{ij} \det(\eta_{ij})$$ (37)

for any $i = 1, \ldots, N-1$ where $h_{ij}$ is the $(i, j)$ in $H$ and $\eta_{ij} \in \mathbb{R}^{(N-2) \times (N-2)}$ is a submatrix with of $H$ with the $i$th row and $j$th column removed[10]. With a fixed $\eta_{ij}$, $\det(H)$ is a linear function of $h_{ij}$, $i = 1, \ldots, N-1$. Considering updating one row vector of $H$ and one entry of $g$ wguke fixing the other rows of $H$ and the other entries of $g$, where $h_i^T$ denote $i$th row vector of $H$ and $g_i$ denote $i$th entry of $g$, partial maximization of (36) wotj respect to $h_i$ and $g_i$ is given by

$$\max_{h_i^T, g_i} \left| \sum_{j=1}^{N-1} (-1)^{i+j} h_{ij} \det(\eta_{ij}) \right|$$ (38a)

With constraint

$$0 \leq h_i^T \tilde{x}[n] - g_i \leq 1 - \sum_{j \neq i} h_{ij}^T \tilde{x}[n] - g_i, \forall n$$ (38b)
Partial maximization can be solved by breaking it into two LPs [4].

\[
p^* = \max_{h^*_i, g_i} \left| \sum_{j=1}^{N-1} (-1)^{i+j} h_{ij} \det(\eta_{ij}) \right|
\]

and

\[
q^* = \min_{h^*_i, g_i} \left| \sum_{j=1}^{N-1} (-1)^{i+j} h_{ij} \det(\eta_{ij}) \right|
\]

Optimal solution of (38) denoted as \((h^*_i, g^*_i)\) is chosen as optimal solution of (39a) if \(|p^*| > |q^*|\) and (39b) if \(|p^*| < |q^*|\). Dimension reduced endmember estimated \(\alpha_1, ..., \alpha_N\) can be achieved by

\[
\alpha = H^{N-1} \ast
\]

\[
[\alpha_1, ..., \alpha_N] = \alpha^T 1_{N-1} + (H^{N-1})
\]

the estimated endmember signature can be recovered by

\[
\tilde{s}[n] = [s'[n]^T \times (1 - 1_{N-1}^T s'[n])]^T
\]

\[
= [(H^* \tilde{x}[n] - g^*)^T \times (1 - 1_{N-1}^T (H^* \tilde{x}[n] - g^*))]^T
\]

V. MVES COMPUTING IMPLEMENTATION AND EXPERIMENTAL RESULT

For implementation of MVES can be simply described as seen on figure 3. Each step on figure 3 is explained in table 1 and heavily referenced on previous section.

We provide two distinct scenario for using MVES to unmixing hyperspectral image. We used sintetic hyperspectral image dataset for first scenario and used AVIRIS real hyperspectral data onto second scenario.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Find parameter ((C, d)), using (13) and (14).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2</td>
<td>obtain ( \tilde{x}[n] = C^* (x[n] - d) ) for any (n).</td>
</tr>
</tbody>
</table>

**Figure 3. MVES Implementation Flowchart**

- **Step 3** Set cyclic descriptor to be used on step 4, \(i=1\) on the mean time set \(g = |\det(H)|\) which will be used to determine the relative change of the conditions used for optimal results and value compared with the tolerance limit for \(10^{-8}\).
  - Obtain feasible initial \((H, g)\) with solving (46)
  - \(\text{find}(H, g)\)
  - \(H\tilde{x}[n] - g \geq 0, 1_{N-1}(H\tilde{x}[n] - g) \leq 1 \forall n = 1, ..., L\)

- **Step 4** From the initial feasible obtained from third step, iterative process that estimates of the number of endmember * 10 iterations and with the restriction
that the values of tolerance relative changes in the optimal solution with previous solutions for more than $10^{-8}$. Operations performed is the start of stage 4 to stage 7. For phase four, is to solve the linear program given in (42a) and (42b) using the barrier function (41b).

In this process needs to make an iteration operation to obtain all the necessary combinations submatrix $H_i$ to compute (42a) and (42b).

Optimal results denoted with $(h_i^T, g_i)$ for maximum $(h_i^T, g_i)$ compute (42a) and (42b).

Step 5: This stage check both value of $p^*$ and $q^*$ which are obtained from (42a) and (42b), if $|p^*| > |q^*|$, then $(h_i^T, g_i) = (h_i^T, g_i)$, otherwise if $|p^*| < |q^*|$, then $(h_i^T, g_i) = (h_i^T, g_i)$.

Step 6: if $(i \mod (N - 1)) \neq 0$, then $i = i + 1$, continue back to step 4, otherwise, recheck:

$$\max\{p^*, |q^*|\} - e < \text{tolerance},$$

if true then $H^* = H$ and $g^* = g$ otherwise $e = \max\{p^*, |q^*|\}$, and

$$i = i + 1,$$ as long as iteration restriction is not passed yet, return to step 4.

Step 7: find $[\hat{\alpha}_1, ..., \hat{\alpha}_{N-1}]$ and $\hat{\alpha}_i$ with calculate (43) and (44). Values of $[\hat{\alpha}_1, ..., \hat{\alpha}_{N-1}]$ and $\hat{\alpha}_i$ will be used to find estimated signature matrix and abundance.

Step 8: find $\hat{\alpha}_i$ using $\hat{\alpha}_i = C \hat{\alpha}_i + d$ for $i = 1, ..., N$ with $\hat{\alpha}_i \in \mathbb{N}^L$.

Continued with finding abundance map using (45).

Table 1. MVES Steps

<table>
<thead>
<tr>
<th>A. Scenario 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using spectral library obtained from [12] and random abundance maps generated with dirichlet distribution we create some synthetic hyperspectral images. We also put on gaussian noise with variance ranging from 0-0.1. Using noised synthetic data produced we operate MVES function to estimate signature matrix to be compared with genuine spectral library obtained from [12] by calculate root mean square of each signatures comparison between original signatures and estimated signatures. The results given in table 2 and described in figure 4.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.72E-08</td>
</tr>
<tr>
<td>0.01</td>
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<td>0.116494</td>
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<tr>
<td>0.09</td>
<td>0.1292</td>
</tr>
<tr>
<td>0.1</td>
<td>0.172211</td>
</tr>
</tbody>
</table>

Table 2. Signature RMS Calculation

Figure 4. RMS Calculation Result Chart
B. Scenario 2

Using dataset obtained from AVIRIS[8], 'j970619t01p02_r02_sc04.a.rfl’ reflectance file. We extracted signature matrix and abundance map using MVES. Estimated signature matrix will be matched manually with spectral library [4] on purpose to set related mineral label onto each abundance map. Result can be seen on Figure 5. Some signature cannot be identified because high dissimilarity between estimated signature matrix and its spectral library.

VI. CONCLUSION

Based on previous experiments we can conclude

1. Minimum-Volume Enclosing Simplex can be used as alternative method to extract endmembers information from hyperspectral image without requiring any ground truth information.

2. In abundance map image, observed endmembers will be marked as a brighter pixel or higher pixel value compared with its surrounding, since abundance is weighting its dominant signature matrix.

Table 3. 14 estimated abundances obtained by MVES: (a) Goethite (b) –undetected (c) Kaolinite#2 (d) Kaolinite#3 (e) pyrope (f) Alunite (g) Goethite #2 (h) Andradite (i) Alunite#2 (j) Muscovite (k) Kaolinite #2 (l) Montmorillonite (m) kaolinite #1 (n) -undetected
3. Manual effort still be required to determine which mineral is related on estimated signature matrix.

VII. REFERENCES


